Design of Mine Working Monolith Lining upon the Action of Underground Waters

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ABSTRACT

The method of designing monolith concrete and ferro-concrete lining of permanent mine workings including those erected with the application of special water suppressions actions upon mine water pressure is given. A complete design alhorithm has been developed and programme for the computer has been created. Design examples are given.

Special measures in water supression consisting in creating hydro-isolation curtains around workings by injecting plug-up solutions into the massif is made as rule in constructing mine workings in complicated hydro-geological conditions characterized by the presence of fissured mining rocks and substantial underwater heads. In this case the working is made in the plugged-up rocks zone, which to a great extent or fully eliminates water inflow both in the period of construction and in the period of the working exploitation. Low water permeability of the plugged-up curtain brings to a redistirbution of the residual head at the underground waters undergoing filtration through the curtain and the working lining. This fact and also the presence of the plugged-up rocks zone having a deformation characteristic different from that of the rest of the massif around the working has a substantial influence upon the lining stressed state, that is why it allows in some cases to lessen the weight of the underground structure by lowering its thickness or the reinforcement coefficient. On account of it in designing mine working linings undergoing the action of underground watere pressure it is worthwile to take into account the presence of the hydro-isolation curtain.

The design methods of lining upon the action of the rock's own weight elaborated at Tula Polytecnical Institute are described in paper[(1)].

The technique of designing a monolith lining of the workings of an arbitrary cross-section (with a single axis symmetry) upon the action of underground water pressure with a possible water filtration through and the lining being taken into account is given below. The technique is based upon the solution of the flat contact problem concerning the equilibrium of a double-laqer noncircular ring the inner layer of which simulates the lining, the outer one simulates the plugged-up rock zone. The designed scheme is given in Fig. 1.

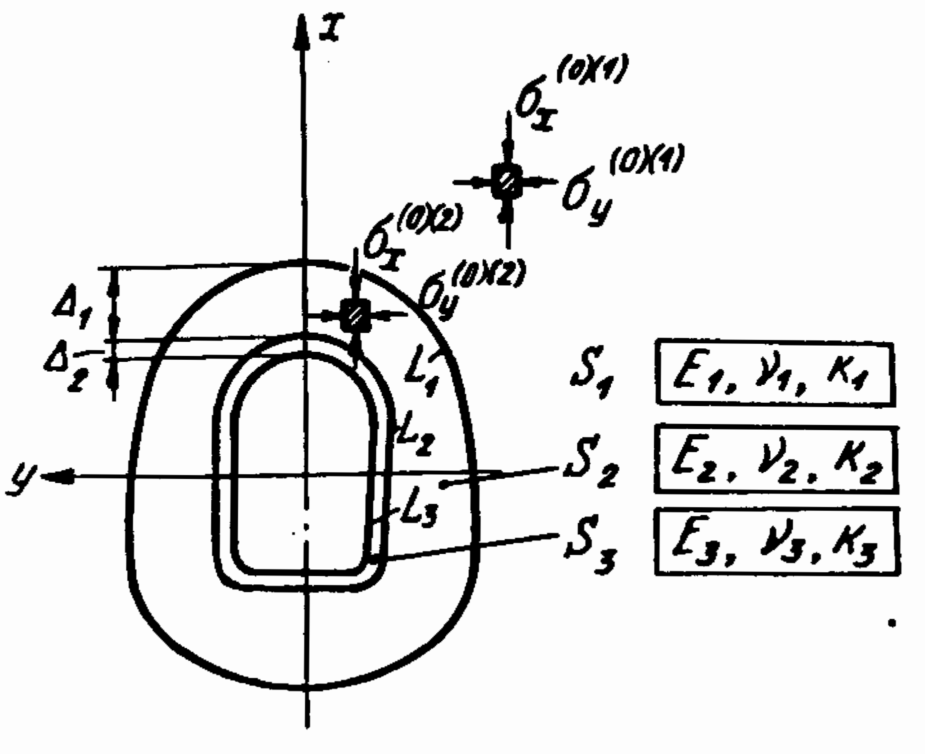


Figure 1. Designed scheme

The medium S_1, simulating the rock massif is characterised by the deformation module E_1, and Poisson ratio ν_1. The outer ring layer S_2 of Δ_1 thickness, the material of which has a E_2 deformation modulus and a ν_2 Poisson ratio simulates the plugged-up rocks zone, the internal layer S_3 of Δ_2 thickness with E_3, ν_3 characteristics being the working lining.

Layer of the ring and the medium undergo deformation together that is conditions of displacement vectors and full stresses continuity are performed upon the L_i $(i = 1,2)$ lines of contact. The inner outline L_3 is free from the actions of external forces.

A uniform underground water pressure (the water pressure change at the height of the strengthened zone is not taken into account which does not introduce substantial errors in the results of the design at sufficient heads) undergo simulation by assigning initial stresses conditioned by residual heads filtering through the plugged-up zone and lining water in fields S_1, S_2 (Fig. 1). Initial stresses are determined in accordance with the standardized document(2) by applying formulae:

$$\sigma_x^{(1)(0)} = \sigma_y^{(1)(0)} = -\gamma_w H_e \frac{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3} + \frac{1}{K_2}\ln\frac{\tilde{R}_1}{\tilde{r}_2}}{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3} + \frac{1}{K_2}\ln\frac{\tilde{R}_1}{\tilde{R}_2} + \frac{1}{K_1}\ln\frac{\tilde{R}_t}{\tilde{R}_1}} \tag{1}$$

- in the sphere S_2

$$\sigma_x^{(2)(0)}=\sigma_y^{(2)(0)}=-\gamma_w H_e \frac{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3}}{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3}+\frac{1}{K_2}\ln\frac{\tilde{R}_1}{\tilde{R}_2}+\frac{1}{K_1}\ln\frac{\tilde{R}_t}{\tilde{R}_1}} \tag{2}$$

where γ_w is the water specific gravity; H_e is the underground water level, being counted off from the beginning of coordinates; R_1, R_2, R_3 are the average radii of the outlines L_1, L_2, L_3 correspondingly; K_1, K_2, K_3 are the coefficients of filtration of the massif, the plugged-up rock's zone and the lining material correspondingly; R_t is the conditional radius of feeding(2).

Registration of tough-elastic deformation of rocks is made on the base of linear heriditory creepage applying the alternative modulii technique(3) according to which rock deformation characteristics being included in the solution of the elasticity theory are given in the time function form.

The problem set forth is solved with the application of the apparatus of the complex variable analitic functions, conformal representations and complex series(4).

Full stresses are given in the form of sums of initial stresses (1), (2) and additional stresses, due to the presence of the working.

On the complex potentials $\varphi_i(\zeta)$, $\psi_i(\zeta)$ (i = 1, 2, 3), regular in corresponding spheres S_i (i = 1, 2, 3) and being turned to zero upon infinite combined with additional stresses and displacements by the Kolosov - Muschelishvili formulae being introduced(4), the contact problem is reduced to the solution of the boundary problem of the complex variable analytic function theory.

With the aid of the rational function of the kind

$$z=\omega(\zeta)=R(\zeta+\sum_{v=1}^{4} q_v\zeta^{-v}) \tag{3}$$

conformal transformation of the radius $R_3 < 1$ circle exterior upon the exterior of the outline L_3 is made in such a way as to make the circumference with the radius $R_1 = 1$ pass into the outline L_1; the radius circumference R_2 ($R_3 < R_2 < R_1$) being transfered into the L_2 outline.

Boundary conditions of the problem set forth in the transformed sphere is written in the following way(5):

$$\bar{\varphi}_i(\frac{R_i}{\sigma})+\frac{\bar{\omega}(\frac{R_i}{\sigma})}{\omega'(R_i\sigma)}\varphi_i'(R_i\sigma)+\psi_i(R_i\sigma)=\bar{\varphi}_{i+1}(\frac{R_i}{\sigma})+\frac{\bar{\omega}(\frac{R_i}{\sigma})}{\omega'(R_i\sigma)}\varphi_{i+1}(R_i\sigma)+\psi_{i+1}(R_i\sigma)+\gamma_w H_e p_i\bar{\omega}(\frac{R_i}{\sigma}) \tag{4}$$

upon L_i (i=1,2)

$$\chi_i\overline{\varphi}_i\left(\frac{R_i}{\sigma}\right)-\frac{\overline{\omega}\left(\frac{R_i}{\sigma}\right)}{\omega'(R_i\sigma)}\varphi_i'(R_i\sigma)+\psi_i(R_i\sigma)=\frac{\mu_i}{\mu_{i+1}}\left[\chi_{i+1}\overline{\varphi}_{i+1}\left(\frac{R_i}{\sigma}\right)-\frac{\overline{\omega}\left(\frac{R_i}{\sigma}\right)}{\omega'(R_i\sigma)}\varphi_{i+1}'(R_i\sigma)+\psi_{i+1}(R_i\sigma)\right]$$

$$\overline{\varphi}_3\left(\frac{R_3}{\sigma}\right)+\frac{\overline{\omega}\left(\frac{R_3}{\sigma}\right)}{\omega'(R_3\sigma)}\varphi_3'(R_3\sigma)+\psi_3(R_3\sigma)=0 \qquad (5)$$

upon L_3

where

$$p_1=\frac{\frac{1}{K_2}\ln\frac{\tilde{R}_1}{\tilde{R}_2}}{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3}+\frac{1}{K_2}\ln\frac{\tilde{R}_1}{\tilde{R}_2}+\frac{1}{K_1}\ln\frac{\tilde{R}_t}{\tilde{R}_1}} \qquad (6)$$

$$p_2=\frac{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3}}{\frac{1}{K_3}\ln\frac{\tilde{R}_2}{\tilde{R}_3}+\frac{1}{K_2}\ln\frac{\tilde{R}_1}{\tilde{R}_2}+\frac{1}{K_3}\ln\frac{\tilde{R}_t}{\tilde{R}_1}};\mu_i=\frac{E_i}{2(1+\nu_i)};\chi_i=3-4\nu_i$$

Complex potentials $\varphi_i(\zeta)$, $\psi_i(\zeta)$, $(i = 1, 2, 3)$ may be represented in the form of complex series

$$\varphi_i(\zeta)=\sum_{\nu=1}^{\infty} c_\nu^{(1)(i)}\zeta^{-\nu}+\sum_{\nu=0}^{\infty} c_\nu^{(3)(i)}\zeta^{\nu}$$

$$\psi_i(\zeta)=\sum_{\nu=1}^{\infty} c_\nu^{(2)(i)}\zeta^{-\nu}+\sum_{\nu=0}^{\infty} c_\nu^{(4)(i)}\zeta^{\nu} \qquad (7)$$

where

$$c_\nu^{(3)(1)} = c_\nu^{(4)(1)} = 0 \qquad (\nu = 1,\ldots,\infty)$$

On account and the geometric and power symmetry concerning the 0_x axis all coefficients of expansion in series are real.

Let us imagine

$$\frac{\bar{\omega}\left(\frac{R_i}{\sigma}\right)}{\omega'(R_i\sigma)}$$

in the form of:

$$\frac{\bar{\omega}\left(\frac{R_i}{\sigma}\right)}{\omega'(R_i\sigma)}=\sum_{K=0}^{n} h_K^{(i)}\sigma^K+\sum_{K=1}^{\infty} h_{-K}^{(i)}\sigma^{-K} \tag{8}$$

Coefficients $h_K^{(i)}$, $h_{-K}^{(i)}$ are determined by recurrence formulae received by dividing polynomial by polynomial[6]:

$$h_{n-k}^{(i)}=R_i^{-(n-k)}q_{n-k}+\delta_{n-k+1}\sum_{\nu=1}^{K-1}(K-\nu)q_{K-\nu}R_i^{\nu-K-1}h_{n-\nu+1}^{(i)} \tag{9}$$

$$(K = 0,...,n)$$

$$k_{-K}^{(i)}=\lambda_{K,1}R_i+\sum_{\nu=1}^{n}\nu q_\nu R_i^{-\nu-1}h_{\nu-K+1}^{(i)}$$

$$(K = 1,...,\infty)$$

where

$$\delta_\nu=\begin{cases}1, \nu<n\\0, \nu\geq n\end{cases}$$

$$\lambda_{i,j}=\begin{cases}1, i=j\\0, i\neq j\end{cases}$$

Substituting expressions (7), (8) in boundary conditiokns (4) upon L_i (i = 1, 2) and equating coefficients in their left and right parts at the same degrees of variable σ, after a series of algebraic transformations we received the correlations combining coefficients $c_\nu^{(j)(2)}$ (j = 1,...,4) expansions in series of potentials of $\varphi_2(\zeta)$, $\psi_2(\zeta)$ with coefficients $c_\nu^{(j)(1)}$ (j = 1, 2) of potentials $\varphi_1(\zeta)$, $\psi_1(\zeta)$, and then the correlations combining coefficients $c_\nu^{(j)(3)}$ (j = 1,...,4) of the expansions of potentials $\varphi_3(\zeta)$, $\psi_3(\zeta)$ with coefficients $c_\nu^{(j)(2)}$ (j = 1,...,4). As a result one is able to express complex potentials $\varphi_3(\nu)$, $\psi_3(\nu)$ expansion coefficients $c_\nu^{(j)(3)}$by formulae

$$c_K^{(j)(3)}=\sum_{m=1}^{4}\sum_{l=1}^{2}\sum_{\nu=1}^{\infty}\sum_{p=1}^{\infty}P_{K,\nu}^{(m,j)(2)}P_{\nu,p}^{(l,m)(1)}c_p^{(l)(1)}+\sum_{m=1}^{4}\sum_{\nu=1}^{\infty}P_{K,\nu}^{(m,j)(2)}Q_\nu^{(m)(1)}+Q_K^{(j)(2)} \tag{10}$$

Here values $P_{r,s}^{(t,f)(i)}$, $Q_r^{(f)(i)}$ (r = 1,...,∞ ; s = 1,...,∞; t = 1,...,4; f = 1,...,4; i = 1, 2) are calculated by formulae:

$$P_{r,s}^{(1,1)(i)}=\lambda_{r,s}\cdot t_i-d_i\cdot R_i^r\cdot a_{r,s}^{(i)}$$

$$P_{r,s}^{(2,1)(i)}=0$$

$$P_{r,s}^{(3,1)(i)}=d_i\cdot R_i^r\cdot a_{r,s}^{\prime(i)}$$

$$P_{r,s}^{(4,1)(i)}=\lambda_{r,s}\cdot d_i\cdot R_i^{2r}$$

$$P_{r,s}^{(1,3)(i)}=-d_i\cdot R_i^{-r}\cdot b_{r,s}^{(i)}$$

$$P_{r,s}^{(2,3)(i)}=\lambda_{r,s}\cdot d_i\cdot R_i^{-2r}$$

$$P_{r,s}^{(3,3)(i)}=\lambda_{r,s}\cdot t_i+d_i\cdot R_i^{-2}\cdot b_{r,s}^{\prime(i)}$$

$$P_{r,s}^{(4,3)(i)}=0$$

$$P_{r,s}^{(1,2)(i)}=R_i^r\sum_{p=1}^{\infty}[b_{r,p}^{(i)}(P_{p,s}^{(1,1)(i)}-\lambda_{s,p}\cdot l_i)-b_{r,p}^{\prime(i)}\cdot P_{p,s}^{(1,3)(i)}]$$

$$P_{r,s}^{(2,2)(i)}=\lambda_{r,s}\cdot l_i+R_i^r\sum_{p=1}^{\infty}(b_{r,p}^{(i)}\cdot P_{p,s}^{(2,1)(i)}-b_{r,p}^{\prime(i)}\cdot P_{p,s}^{(2,3)(i)}) \tag{11}$$

$$P_{r,s}^{(3,2)(i)}=\lambda_{r,s}\cdot S_i R_i^{2r}+R_i^r\sum_{p=1}^{\infty}[b_{r,p}^{(i)}\cdot P_{p,s}^{(3,1)(i)}-b_{r,p}^{\prime(i)}(P_{p,s}^{(3,3)(i)}-\lambda_{s,p}\cdot l_i)]$$

$$P_{r,s}^{(4,2)(i)}=R_i^r\sum_{p=1}^{\infty}b_{r,p}^{(i)}\cdot P_{p,s}^{(4,1)(i)}$$

$$P_{r,s}^{(1,4)(i)}=\lambda_{r,s}\cdot S_i\cdot R_i^{-2r}+R_i^{-r}\sum_{p=1}^{\infty}[a_{r,p}^{(i)}(P_{p,s}^{(1,1)(i)}-\lambda_{s,p}\cdot l_i)-a_{r,p}^{\prime(i)}\cdot P_{p,s}^{(1,3)(i)}]$$

$$P_{r,s}^{(2,4)(i)}=-R_i^{-r}\sum_{p=1}^{\infty}a_{r,p}^{\prime(i)}\cdot P_{p,s}^{(2,3)(i)}$$

$$P_{r,s}^{(3,4)(i)}=R_i^{-r}\sum_{p=1}^{\infty}[a_{r,p}^{(i)}\cdot P_{p,s}^{(3,1)(i)}-a_{r,p}^{\prime(i)}(P_{p,s}^{(3,3)(i)}-\lambda_{p,s}\cdot l_i)]$$

$$P_{r,s}^{(4,4)(i)}=\lambda_{r,s}\cdot l_i+R_i^{-r}\sum_{p=1}^{\infty} a_{r,p}^{(i)}\cdot P_{p,s}^{(4,1)(i)}$$

where

$$\beta_{i+1}=\frac{\mu_{i+1}}{\mu_i}$$

$$t_i=\frac{1+\chi_i\cdot\beta_{i+1}}{1+\chi_{i+1}}$$

$$d_i=\frac{1-\beta_{i+1}}{1+\chi_{i+1}}$$

$$l_i=1-d_i$$

$$s_i=1-t_i$$

$$a_{K,\nu}^{(i)}=\delta_{\nu+K}\cdot\nu\cdot h_{\nu+K+1}\cdot R_i^{-\nu-1}$$

$$a_{K,\nu}^{\prime(i)}=\delta_{K-\nu}\cdot\nu\cdot h_{K-\nu+1}^{(i)}\cdot R_i^{\nu-1}$$

$$b_{K,\nu}^{(i)}=\delta_{\nu-K}\cdot\nu\cdot h_{\nu-K+1}\cdot R_i^{-\nu-1}$$

$$b_{K,\nu}^{\prime(i)}=\nu\cdot h_{-K-\nu+1}^{(i)}\cdot R_i^{\nu-1}$$

($i = 1,2,3$; $K = 0,\ldots,\infty$; $\nu = 1,\ldots,\infty$)

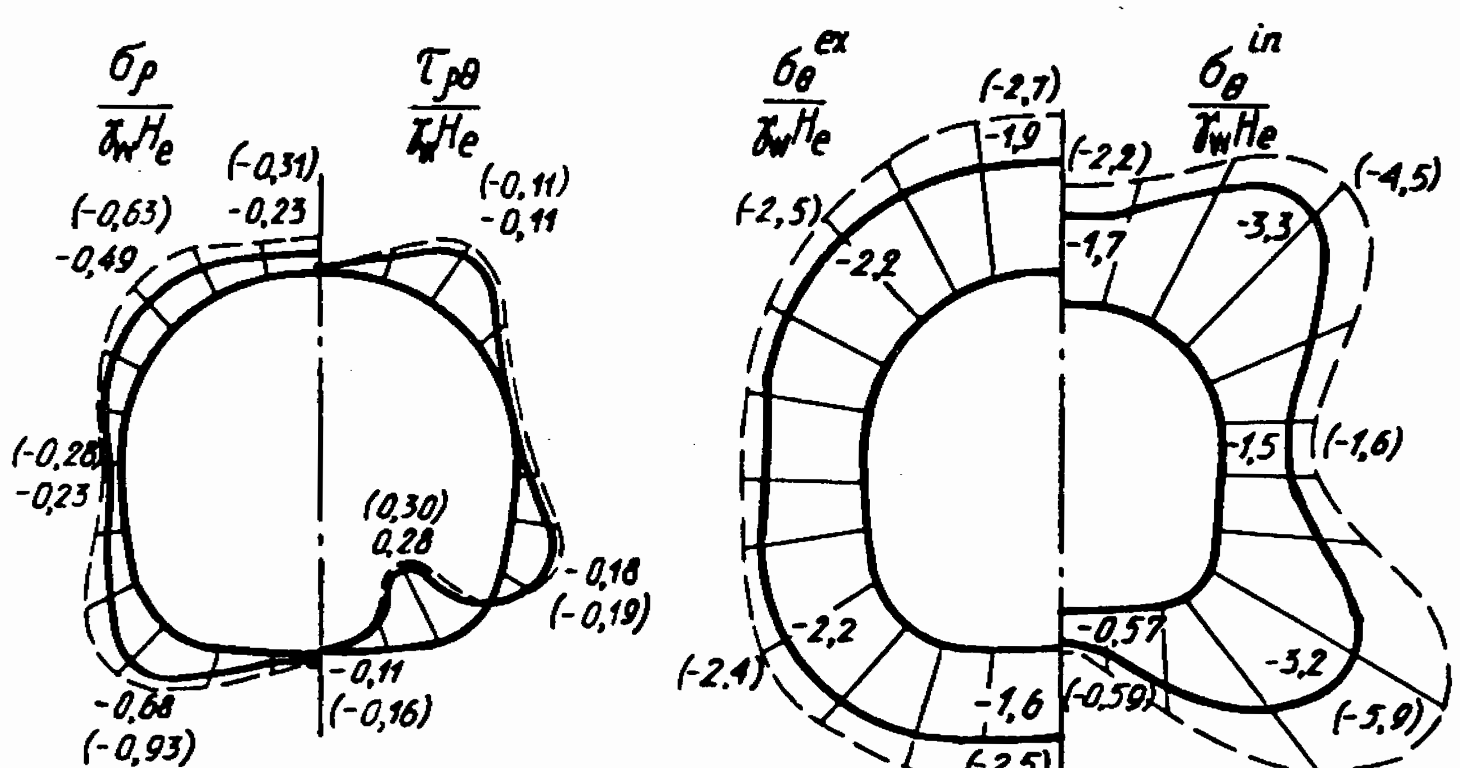
Expressions (10) for coefficients $c_K^{(j)(3)}$ of expansion of potentials $\varphi_3(\zeta)$, $\psi_3(\zeta)$ are substituted in boundary condition (5) upon the outline L_3, from which an infinite system of linear algebraic equations relative to the unknown ones is obtained on the coefficients being equeted at equel degrees of variables. Coefficients $c_K^{(j)(i)}$ ($j = 1,\ldots,4$; $i = 2, 3$) are determined on the basis of the correlations received (10) upon solving the system being correspondingly shorted to 60 equations ($K = 1,\ldots,30$) which secures an accracy sufficient for practical calculation, while stresses and displacements in spheres S_i ($i = 1, 2, 3$) are determined by the Kolosov - Muschelishvili formulae. Initial stresses (1), (2) are supplemented to the additional stresses in spheres S_1, S_2.

A complete calculation alhorithm has been developed on the basis of the solution received. The computer programme has been created. The programme allows the calculation processs to be fully automatized. Labour consumption of the basic data preparation not being great and the design time of one variant taking only a few minutes, the programme developed may successfully be applied at practically multi-variant designing.

Results of designing the lining of mine working of an arched outline with a 3 m span and a 2.8 m height upon the action of underground water pressure are given as an illustration below.

The following initial data were taken into account: pluggedup rocks layer thickness $\Delta_1 = 1.5$ m; lining thickness $\Delta_2 = 0.2$ m; deformation and filtrated characteristics of the massif; plugged-up rocks and the lining material $E_1 = 4800$ MPa, $\nu_1 = 0.3$, $K_1 = 0.19x10^{-5}$ m/c; $E_2 = 9600$ MPa, $\nu_2 = 0.3$, $K_2 = 0.5x10^{-6}$ m/c; $E_3 = 24000$ MPa, $\nu_3 = 0.2$, $K_3 = 1.83x10^{-8}$ m/c correspondingly.

Results of the design are given in Fig. 2 a, b in the form of epures of normal $\sigma_\varrho/\gamma_w H_e$ and tangential $\tau_{\varrho\theta}/\gamma_w H_e$ stresses upon the contact of the lining with plugged-up rocks (Fig. 2a), normal-tangential stresses upon external $\sigma_\theta^{ex}/\gamma w.H_e$ and internal $\sigma_\theta^{in}/\gamma_w H_e$ outlines of the lining cross-section (Fig. 2b). For comparison epures of the same stresses (numerical values are given in brackets) without the plugged-up curtain are shown in dotted lines.



a) b)

Figure 2. Design stress epures in lining under the external pressure of underground waters

The results given allows us to make a conclusion that the rocks layer with a low filtration coefficient and elivated deformation modulus value formed with the aid of plugging-up secures a stress decrease in the lining. Thus in the example given normal tangential stresses upon the internal lining cross-section outline undergo an average 23 % decrease, upon the external outline undergoes an 14 %.

One must mark, the presence of plugged-up rocks being created with the aim of water suppression does not allow to bring to an decrease of stresses in lining, called forth by underground water pressure. Thus, for a particular case when the workinglining and the plugged-up zone have a circular form in the cross-section it is not difficult to be convinced having investigated a closed up expression for normal tangential stresses upon an internal outline of lining cross-section, having the form(6) upon extremum

$$\sigma_\theta = -2\gamma_w \cdot H_e \frac{2c_0(1-\nu_2)\cdot\beta\, p_1 + (A + a_2\cdot\beta)\, p_2}{\beta^2 \cdot B(c_0-1) + \beta\, c + A(1-c_2)} \tag{12}$$

where

$$c_0 = \left(\frac{\tilde{R}_1}{\tilde{R}_2}\right)^2$$

$$c_2 = \left(\frac{\tilde{R}_3}{\tilde{R}_1}\right)^2$$

$$a_2 = 1 - 2\nu_2 + c_0$$

$$a_1 = (1-2\nu_2)c_0 + 1$$

$$A = (1-2\nu_2)(c_0-1)$$

$$B = (1-2\nu_3+c_2)\frac{\mu_1}{\mu_3}$$

$$\beta = \frac{\mu_2}{\mu_1}$$

$$C = a_2(1-c_2) + a_1 B$$

p_1 and p_2 are determined by formulae (6).

Having brought derivation from expression (12) to zero at and having made a corresponding analysis we find out that extreme (maximum) stresses σ_θ^{in} in lining arise at values

$$\beta = -M + \sqrt{M^2 - L} \tag{13}$$

where

$$M=\frac{A\cdot p_2}{2c_0(1-\nu_1)p_1+a_2p_2}$$

$$L=\frac{A[a_1Bp_2-2c_0(1-c_2)(1-\nu_2)p_1]}{B(c_0-1)[a_2p_2+2c_0(1-\nu_2)p_1]} \tag{14}$$

Here the condition of tghe extremum existing at β is the fulfilment of non-equality

$$\frac{p_1}{p_2}<\frac{2c_0(1-c_2)(1-\nu_2)}{a_1B} \tag{15}$$

Therefore, the case when filtration coefficient decrease and a plugged-up zone rock deformation modulus increase brings not only to a decrease, but to an increase of stress in lining is not excluded.

It is impossible for correlations of the (12), (13), (15) type to be received analytically for linings of non-circular cross-section as the solution of the problem has no confined form. That is why investigation of the dependence of stresses $\sigma_\theta^{in}/\ \gamma_w H_e$ in a arched shape lining upon the plugged-up zone rocks deformation modulus increase rate has to be made with the aid of multi-variant calculations.

In conclusion we are to mark that the computer elaborated programme functions in the programme complex meant for designing underground structures of different purposses upon static loads, i.e. upon the rocks own weight, underground water pressure, internal head, tectonic forces in rock massif and also upon the seismic earthquake influence have been elaborated.

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