MINE WATER. GRANADA, SPAIN. 1985.

A GAME THEORETICAL MODEL FOR THE EXAMINATION OF THE MUTUAL EFFECT OF MINING AND ENVIRONMENTAL CONTROL

Sandor Molnár[1] and Ferenc Szidarovszky[2]

[1]Dept. head, Central Mining Development Institute, Budapest, III. Mikoviny u. 2/4, H-1037.

[2]Dept. head, Dept. of Math. and Comp. Sci., University of Horticulture, Budapest, XI. Villányi ut 29/35, H-1118.

ABSTRACT

The purpose of this paper is to demonstrate how a multiobjective programming model representing an aquifer management problem can be solved by the theory and methods of noncooperative games. As a case study, a regional groundwater system in Hungary is investigated, in which mining, water supply and environmental protection are the objectives.

1. INTRODUCTION

Large-scale coal mining development is being planned in the region of the Transdanubian Mountains, Hungary. However, mineral resources are located below karstic aquifer water level. At the same time, water supply of the region is provided from this karstic aquifer to meet rapidly growing municipal and industrial water demands. The sinking of the karstic water level is adverse from an environmental point of view because the natural recharge of the thermal waters of Budapest and Heviz comes from this karstic aquifer.

The main idea of this study is to provide a conjunctive analyses of regional mineral resource exploitation, water supply and environmental protection. These three viewpoints are described by three objective functions which cannot be expressed in commensurable units. Furthermore, a solution point which yields an optimal value for one objective usually yields far-from-optimal values of the other objectives. As a result, we must consider this optimization problem as a three-objective problem with three decision makers, since mining, water supply and environmental protection are controlled by different authorities. This problem will be solved as a three-person noncooperative game, in which the decision makers are the "players" and the objective functions are the "payoff" functions.

In the application section, a short presentation of the mathematical model is followed by numerical results.

2. THE MATHEMATICAL MODEL

In this section, the mathematical model of the three-objective management problem of an aquifer as a three-person game is described. First the objectives and constraints are specified.

The principal mine water control problem is caused by the karstic aquifer whose depth is several hundreds of meters and which is located just under the coal deposits. An appropriate combination of mine water control techniques should be found for the planned mines. The efficiency of mine water control depends on the cost of the control and the amount of withdrawal. The higher the efficiency of mine water control, the lower the cost of mining production.

The karstic aquifer is considered to be the most important regional water supply source for the region under consideration. The aquifer is presently being pumped at some existing mines to supply drinking water and to meet industrial water demand. As a result of regional industrialization, both municipal and industrial water demands are growing. A high percentage of the safe yield of karstic water appears to be necessary to meet water demand.

The karstic region under consideration provides the natural recharge of Budapest and Heviz thermal waters. According to numerous studies it is the rainfed natural recharge, 30 m^3/min on the average, which activates various springs and wells feeding the spas and thermal baths. More precisely, a certain portion of the rainfall infiltrates, and then, leaves the system to reappear as thermal water. Any mining development scheme should strive to maintain the existing discharge and the water quality of thermal baths. This requirement is the environmental protection objective of the problem.

The elements of the problem are next described and formulated in terms of decision variables, objective functions and constraints.

Let the water withdrawal sites be denoted by index $i = 1,2,\dots,n$, with n_1 mines, n_2 other sites /wells/ and let the artificial recharge points be denoted by index $k = 1,\dots,r$. Denote $n = n_1 + n_2$.

The decision variables are defined as follows:

x_i = yield of withdrawal from mine i;

m_i = yield of inrush, that is, underground water entry allowed into mine i;

d_i = yield inrush allowed into the inner drainage system of mine i by using instantan protection;

g_i = yield of water prevented from entering mine i by sealing or grouting;

v_{ik} = yield of water conveyed from mine i to recharge point k;

y_{ij} = annual amount of water supplied from mine or other intake i to water demand grid point j.

The economic function are, all in annual values, as follows:

$C_i^1(x_i)$ = costs of withdrawal from mine i;

$L_i(m_i)$ = economic loss due to the occurrence of inrushes m_i;

$C_i^2(d_i)$ = costs of inner drainage;

$C_i^3(g_i)$ = sealing cost;

$C_{ik}^4(v_{ik})$ = conveyance cost.

By using this notation, the cost function of mine i is given as

$$f_{1i} = C_i^1(x_i) + L_i(m_i) + C_i^2(d_i) + C_i^3(g_i) + \sum_{k=1}^{r} C_{ik}^4(v_{ik}). \qquad /1/$$

The regional maining objective is to minimize the sum of water related costs, which is a traditional objective in the mining industry.

The constraints are defined for i=1,...,n as follows:

$$m_i + d_i = x_i$$
$$x_i + g_i = A_i \qquad /2/$$

where

A_i = average yield of inrush into mine i in the absence of input control.

Thus the overall mining objective is

$$\text{Min } f_1 = \sum_i f_{1i} \qquad /3/$$

or by introducing the first payoff function $\Phi_1 = -f_1$,

$$\text{Max } \Phi_1, \qquad /4/$$

since the payoff functions are to be maximized.

Regional water management aims at satisfying water demands at the least feasible cost. Possible groundwater intakes /including mines/ and water demands are considered at grid points over the region. Then the water-supply objective can be expressed as

$$\text{Min } f_2 = \sum_{i=1}^{n} \sum_{j=1}^{m} s_{ij}\, y_{ij}\ , \qquad /5/$$

or equivalently

$$\text{Max } \Phi_2, \qquad /6/$$

where $\Phi_2 = -f_2$ is the second payoff function, furthermore, let

M = the number of water demand grid points;

D_j = water demand at grid point j j=1,2,...,m ;

S_{ij} = annual cost of supply including capital, operation treatment, and conveyance costs.

The following mass balance constraints should be also considered:

$$\sum_{i=1}^{n} y_{ij} = D_j; \text{ for } j = 1,2,\ldots,M \qquad /7/$$

$$\sum_{j=1}^{m} y_{ij} + \sum_{k=1}^{r} v_{ik} \leqq x_i \text{ for every mine } i=1,2,\ldots,n_1.$$

The environmental objective is satisfied if the decisions $\{x_i, y_{ij}, v_{ik}\}$ are realized such that the amount of system outflow q necessary for the recharge of thermal waters is maximized. Thus, the environmental objective can be formulated as:

$$\text{Max } \Phi_3 = q(\underline{x}, \underline{y}, \underline{v}) \qquad /8/$$

However, exceeding the present value of the discharge $q_0 = 30$ m³/min may be detrimental to certain qualitative characteristics of the thermal spas. Thus, the following constraint is added:

$$q \leqq q_0 \qquad /9/$$

Function q is estimated by means of the regional karstic water model /based on a system of partial differencial equations/. The calculations can be approached in the following way. In order to calibrate Eq.8, a random sample values $(\underline{x}, \underline{y}, \underline{v})$ is selected, the regional water model is then used to calculate the corresponding values of the discharge q. These calculated values of q are fitted by least squares to a linear function of the decision variables. Thus the third payoff function Φ_3 is considered to be linear.

3. THE SOLUTION METHOD

Observe that the decision variables can be devided into three groups: $\underline{u}_1 = (x_i, d_i)$ characterizes the mining objective, $\underline{u}_2 = (y_{ij})$ is the decision vector for the water supply objective, and $\underline{u}_3 = (v_{ik})$ gives the decision water for the environmental objective.

Assume furthermore, that all functions $C_i^1 x_i$, $L_i(x_i - d_i)$, $C_i^2(d_i)$, $C_i^3(A_i - x_i)$, $C_{ik}^4(v_{ik})$, $S_{ij}(y_{ij})$ are linear. Then, since all constraints /2/, /8/ and /9/ are also linear, we may assume that the payoff functions have the linear form

$$\Phi_i = \underline{c}_{i1}^T \underline{u}_1 + \underline{c}_{i2}^T \underline{u}_2 + \underline{c}_{i3}^T \underline{u}_3, \quad (i=1,2,3) \qquad /10/$$

where $\underline{u} = (\underline{u}_1, \underline{u}_2, \underline{u}_3)$ and the set of constraints can be summarized as

$$\underline{\underline{A}}_1 \underline{u}_1 + \underline{\underline{A}}_2 \underline{u}_2 + \underline{\underline{A}}_3 \underline{u}_3 \geq \underline{b}. \qquad /11/$$

That is, the set of simultaneous strategies is given by the constraint /11/.

Assume that $(\underline{u}_1^*, \underline{u}_2^*, \underline{u}_3^*)$ is a Nash-equilibrium point /see Szidarovszky, 1978/ of the above defined three-person noncooperative game, then for i=1,2,3, $\underline{u}_i^*$ is a maximum point of Φ_i with fixed $\underline{u}_j^*$ - s /j≠i/. Thus, the Kuhn-Tucker conditions /see Hadley, 1964/ imply that

$$\begin{aligned} &\underline{c}_{kk}^T + \underline{v}_k^{*T} \underline{\underline{A}}_k = \underline{o}^T, \\ &\underline{\underline{A}}_1 \underline{u}_1^* + \underline{\underline{A}}_2 \underline{u}_2^* + \underline{\underline{A}}_3 \underline{u}_3^* \geq \underline{b}, \\ &\underline{v}_k^{*T} (\underline{\underline{A}}_1 \underline{u}_1^* + \underline{\underline{A}}_2 \underline{u}_2^* + \underline{\underline{A}}_3 \underline{u}_3^* - \underline{b}) = 0 \\ &\qquad \underline{v}_k^* \geq \underline{o} \end{aligned} \qquad /12/$$

for all k=1,2,3. Observe that the left hand side of the third constraint is nonnegative for any feasible solution, consequently conditions /12/ are equivalent to the optimization problem

$$\left.\begin{aligned} \underline{v}_k &\geq \underline{o} \\ \underline{\underline{A}}_k^T \underline{v}_k &= -\underline{c}_{kk} \end{aligned}\right\} \quad k=1,2,3 \qquad /13/$$

$$\underline{\underline{A}}_1 \underline{u}_1 + \underline{\underline{A}}_2 \underline{u}_2 + \underline{\underline{A}}_3 \underline{u}_3 \geq \underline{b}$$

$$\sum_{k=1}^{3} \underline{v}_k^T (\underline{\underline{A}}_1 \underline{u}_1 + \underline{\underline{A}}_2 \underline{u}_2 + \underline{\underline{A}}_3 \underline{u}_3 - \underline{b}) \longrightarrow \min,$$

where vectors $\underline{u}_k$, $\underline{v}_k$ $(k=1,2,3)$ are the only unknowns. By using the first conditions of /12/, this objective function can be reduced as

$$\sum_{i \neq k} \underline{v}_k^T \underline{\underline{A}}_i \underline{u}_i - \sum_{k=1}^{3} \underline{c}_{kk}^T \underline{u}_k - \sum_{k=1}^{3} \underline{b}^T \underline{v}_k. \qquad /14/$$

Introduce the notations

$$\underline{\underline{H}} = \begin{pmatrix} \underline{\underline{O}} & \underline{\underline{A}}_2 & \underline{\underline{A}}_3 \\ \underline{\underline{A}}_1 & \underline{\underline{O}} & \underline{\underline{A}}_3 \\ \underline{\underline{A}}_1 & \underline{\underline{A}}_2 & \underline{\underline{O}} \end{pmatrix}, \qquad \underline{c} = \begin{pmatrix} \underline{c}_{11} \\ \underline{c}_{22} \\ \underline{c}_{33} \end{pmatrix}, \qquad \underline{B} = \begin{pmatrix} \underline{b} \\ \underline{b} \\ \underline{b} \end{pmatrix}, \qquad /15/$$

$$\underline{u} = \begin{pmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \end{pmatrix}, \qquad \underline{v} = \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \underline{v}_3 \end{pmatrix},$$

then function /14/ can be rewritten as

$$\underline{v}^T \underline{\underline{H}}\, \underline{u} - \underline{c}^T \underline{u} - \underline{B}^T \underline{v}. \qquad /16/$$

Thus the solution of the three-person linear game defined above can be obtained by solving the quadratic programming problem /13/ with modified objective function /16/.

4. NUMERICAL EXAMPLE

The case study introduced by Duckstein at all /1979/ and solved by compromise programming has been investigated by using also the methodology of the previous section. A cooperative game theoretical solution of the same model is also described in Szidarovszky at all /1984/.

The regression model for obtaining function q is obtained as

$$\begin{aligned} q = 30.5 &- 0.021x_1 - 0.012x_2 - 0.014x_4 \\ &- 0.006y_4 - 0.0021y_5 - 0.0042y_6 \qquad /17/ \\ &+ 0.07v_1 + 0.14v_2 \end{aligned}$$

in which $y_i = \sum y_{ij}$ and $v_k = v_{ik}$. The standard error of estimate of the fit in Eq /17/ is 0.15 m^3/min; consequently it appears that q can be estimated with reasonable accuracy as a function of withdrawals $\underline{x}$, $\underline{y}$ and artificial recharge $\underline{v}$ by a linear function.

Three mines are considered with A_1 = 60 m^3/min, A_2 = 150 m^3/min, A_3 = 100 m^3/min. The annual investment and operating costs are given in Table 1. Besides the three mines six water with drawal sites, and seven users are considered. The annual investment and operation costs for water conveyance are given in Table 2 /in unit 10^4 Ft/m^3.min^{-1}/. The water demand, of the users are 33, 14, 83, 14, 14, 83, 28 m^3/min, respectively. Two artificial recharge points are considered, their costs are presented in Table 3.

The quadratic programming problem has been solved by the method of Wolfe /see Kreko, 1972/. The numerical results are summarized in Table 4 and 5.

i	withdrawal			instantan protection		sealing or grouting	
	invest-ment	opera-tion	loss	invest-ment	opera-tion	invest-ment	opera-tion
1	28.5	73.5	940	not feasible		12	42
2	32.0	116	1200	36	129	8.5	25.5
3	100.0	184	1420	45	147	22.3	34.7

Table 1

Investment and Operation Costs and Losses

i	1		2		3		4		5		6		7	
1	6	3	7	8	9	7	11	8	14	8	15	7	17	7
2	9	10	5	4	5	2	8	4	10	4	10	4	14	6
3	9	11	5	2	7	3	10	12	10	6	14	8	18	9
4	261	45	257	40	255	39	256	39	262	43	264	45	268	46
5	272	45	269	43	262	40	259	40	259	30	253	37	256	40
6	264	42	258	32	261	35	265	57	258	32	262	32	265	34

Table 2

Water Conveyance Costs

i	Artificial recharge sites 1.		2.	
1	25	0	45	53
2	68	112	72	78
3	63	94	50	32

Table 3

Costs for Artificial Recharge Sites

i	x_i	m_i	d_i	g_i	v_{i1}	v_{i2}
1	33	33	0	27	0	0
2	150	0	150	0	0	0
3	100	0	100	0	0	25,16

Table 4

Optimal Values of x_i, m_i, d_i, g_i, v_{i1}, v_{i2}

i	y_{i1}	y_{i2}	y_{i3}	y_{i4}	y_{i5}	y_{i6}	y_{i7}
1	33	0	0	0	0	0	0
2	0	0	25	14	0	83	28
3	0	14	58	0	2,84	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	11,15	0	0
6	0	0	0	0	0	0	0

Table 5

Optimal Values of y_{ij}

As a comparison to the numerical results obtained by using other methods /as compromise programming in Duckstein at all., 1979; and cooperative games as reported in Szidarovszky at all., 1984/ we immidiately observe that the use of noncooperative games results in different solutions. This preparty is not a surprise, since noncooperative games reflect a special behaviour of the decision makers, in which no cooperation, no compromise is assumed among them. Since these results are much more unfavourable than those obtained by either cooperative games or compromise programming this case study gives an evidence for the need of smart compromise or cooperation rather than to strive for conflict situations.

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